EXERCISES [MAI 4.8]

PROBABILITY II (TREE DIAGRAMS)

SOLUTIONS

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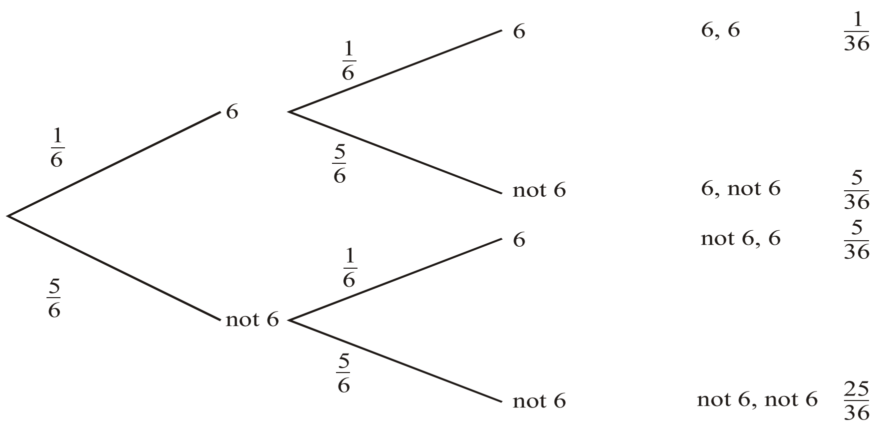
A. Paper 1 questions (SHORT)

1. (a)



(b) P(two sixes) = $\dfrac{1}{36}$

(c) P(one six only) = $\dfrac{5}{36}+\dfrac{5}{36}=\dfrac{10}{36}$

(d) P(one or more sixes) = $\dfrac{1}{36}+\dfrac{10}{36}=\dfrac{11}{36}$ **or** $1-\dfrac{25}{36}=\dfrac{11}{36}$

2. The four scenarios have probabilities 0.18, 0.12, 0.14, 0.56

(a)

$P(A)$	0.3	$P(A')$	0.7	$P(A\cap B)$	0.18
$P(A\cap B')$	0.12	$P(A'\cap B)$	0.14	$P(A'\cap B')$	0.56

(b)

$P(B)$	0.32	$P(B')$	0.68	$P(A\cup B)$	0.44

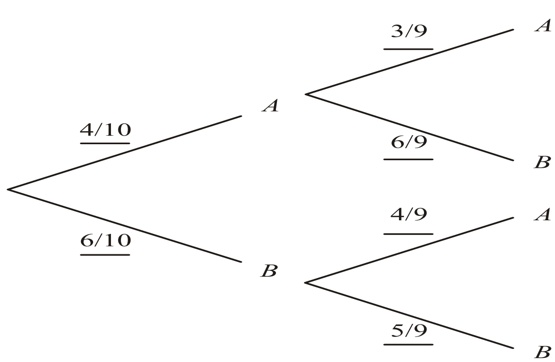
(c)

$P(B\mid A)$	0.6	$P(B'\mid A)$	0.4	$P(B\mid A')$	0.2	$P(B'\mid A')$	0.8

(d)

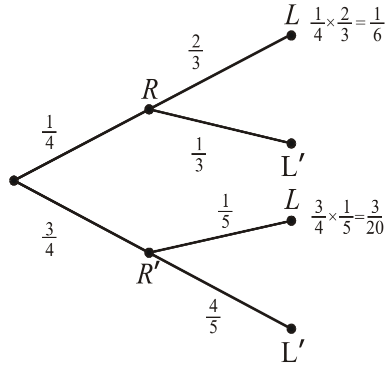
$P(A\mid B)$	0.18/0.32 = 9/16

3. (a)



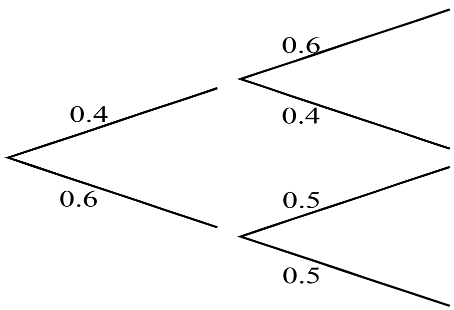
(b) $\left(\dfrac{4}{10}\times\dfrac{6}{9}\right)+\left(\dfrac{6}{10}\times\dfrac{4}{9}\right)=\dfrac{48}{90}\ \left(\dfrac{8}{15},0.533\right)$

4. Let $P(R|L)$ be the probability that it is raining given that the girl is late.



$$P(R|L) = \frac{P(R \cap L)}{P(L)} = \frac{1/6}{1/6 + 3/20} = \frac{10}{19}$$

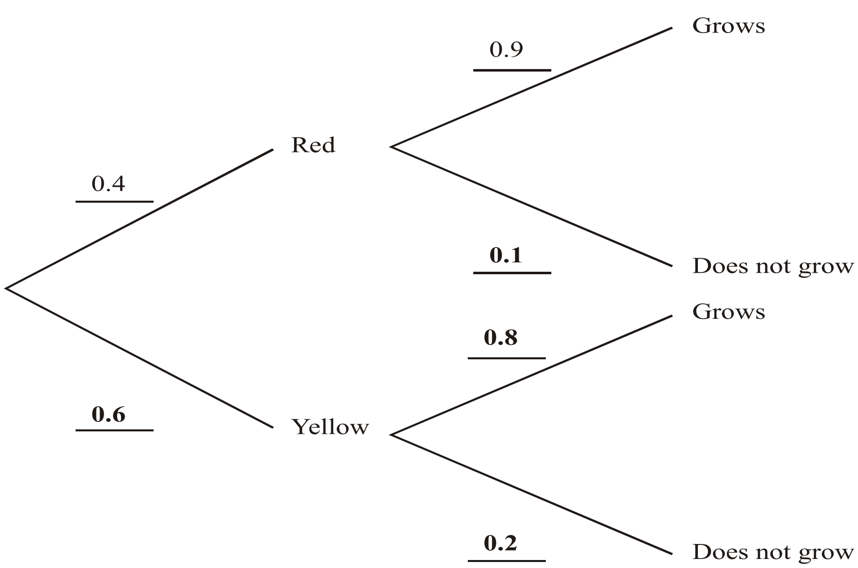
5. (a)



(b) $P(B) = 0.4(0.6) + 0.6\,(0.5) = 0.24 + 0.30 = 0.54$

(c) $P(C|B) = \frac{P(B \cap C)}{P(B)} = \frac{0.24}{0.54} = \frac{4}{9}$ (= 0.444, 3 sf)

6. (a)

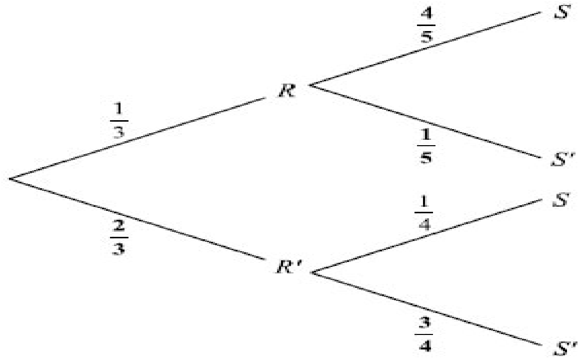


(b) (i) $0.4 \times 0.9 = 0.36$

(ii) $0.36 + 0.6 \times 0.8 \quad (= 0.36 + 0.48) = 0.84$

(iii) $\frac{P(\text{red} \cap \text{grows})}{P(\text{grows})}$ (may be implied) $= \frac{0.36}{0.84} = 0.429 \left(\frac{3}{7}\right)$

7. (a)



(b) (i) $P(R \cap S) = \frac{1}{3} \times \frac{4}{5} \left(= \frac{4}{15} = 0.267 \right)$

(ii) $P(S) = \frac{1}{3} \times \frac{4}{5} + \frac{2}{3} \times \frac{1}{4} = \frac{13}{30}$ (= 0.433)

(iii) $P(R \mid S) =$ (4/15) / (13/30) = $\frac{8}{13}$ (= 0.615)

8. (a) $p = \frac{4}{5}$

(b) $P(B) = \frac{1}{5} \times \frac{1}{4} + \frac{4}{5} \times \frac{3}{8} = \frac{14}{40} \left(= \frac{7}{20} \right)$

(c) $P(A' \mid B) = \frac{\frac{4}{5} \times \frac{3}{8}}{\frac{7}{20}} = \frac{12}{14} \left(= \frac{6}{7} \right)$

9. (a) P(pass) = 0.6 × 0.8 + 0.4 × 0.9 = 0.84

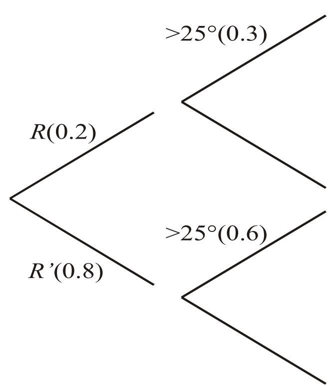
(b) $P(B) = x$, $P(A) = 1 - x$

$0.8(1 - x) + 0.9x = 0.87 \Leftrightarrow x = 0.7$ 70 % from B

10. (a) P(win) = (0.65)(0.83) + (0.35)(0.26) = 0.6305 (or 0.631)

(b) $P(H \mid W') = \frac{(0.65)(0.17)}{0.3695} \left(= \frac{0.1105}{0.3695} \right) = 0.299$

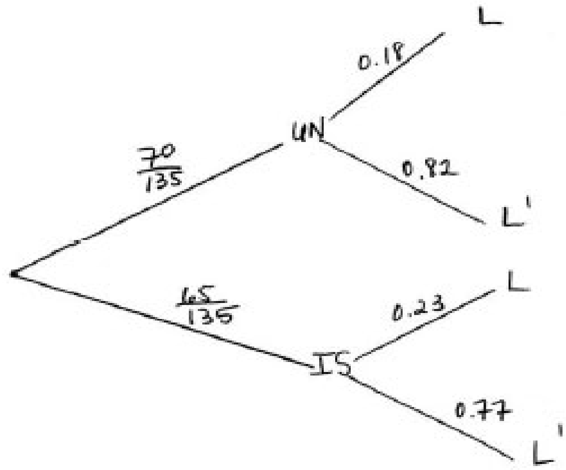
11.



P (> 25°) = 0.2 × 0.3 + 0.8 × 0.6 = 0.54

$P(R \mid >25°) = \frac{0.06}{0.54} = \frac{1}{9}$ (or 0.111)

12. **METHOD 1**



$$P(I \mid L) = \frac{0.23 \times \frac{65}{135}}{0.18 \times \frac{70}{135} + 0.23 \times \frac{65}{135}} = \frac{299}{551} \ (= 0.543, \text{accept } 0.542)$$

METHOD 2

Expected number of suitcases lost by UN Air is $0.18 \times 70 = 12.6$

Expected number of suitcases lost by IS Air is $0.23 \times 65 = 14.95$

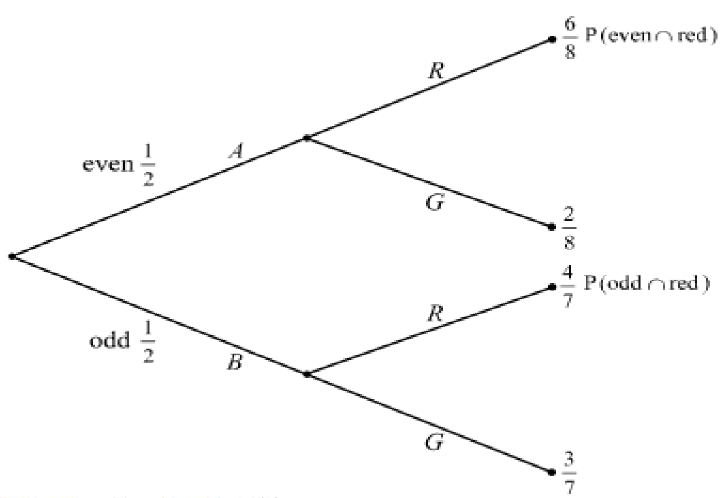
$$P(I \mid L) = \frac{14.95}{12.6 + 14.95} = 0.543$$

13. (a) Probability = $0.2 \times 0.66 + 0.8 \times 0.75 = 0.732$

(b) Probability = $\frac{P(\text{Mon} \cap \text{catches train})}{P(\text{catches train})} = \frac{0.2 \times 0.66}{0.732} = 0.180 \left(= \frac{11}{61}\right)$

14. Required prob = $\frac{3}{9} \times \frac{2}{5} + \frac{6}{9} \times \frac{3}{4} = \frac{19}{30}$

15.



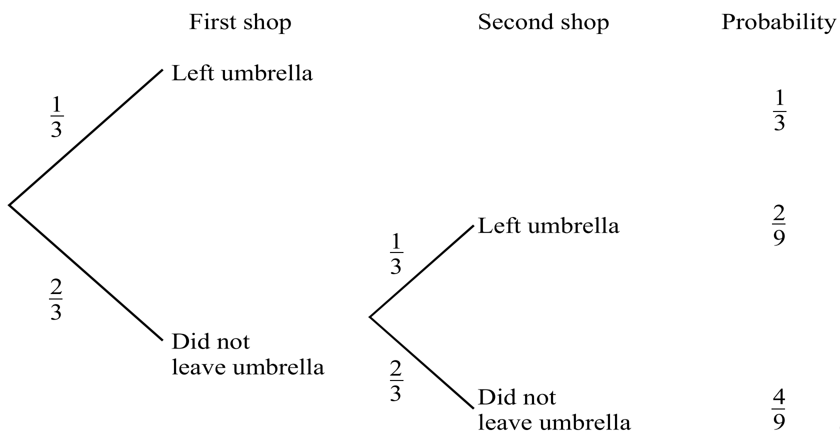
(i) $P(R) = \frac{1}{2} \times \frac{6}{8} + \frac{1}{2} \times \frac{4}{7} = \frac{3}{8} + \frac{2}{7} = \frac{37}{56}$ (0.661)

(ii) $P(B \mid R) = \frac{P(B \cap R)}{P(R)} = \frac{\frac{2}{7}}{\frac{37}{56}} = \frac{16}{37}$ (0.432)

16. (a) $P(\text{same colour}) = \left(\frac{4}{9}\right)\left(\frac{7}{9}\right)+\left(\frac{5}{9}\right)\left(\frac{2}{9}\right) = \left(\frac{28}{81}\right)+\left(\frac{10}{81}\right)=\frac{38}{81} \quad (=0.469)$

(b) $P(\text{first red}|\text{different}) = \dfrac{\left(\frac{4}{9}\right)\left(\frac{2}{9}\right)}{\left(\frac{4}{9}\right)\left(\frac{2}{9}\right)+\left(\frac{5}{9}\right)\left(\frac{7}{9}\right)} = \frac{8}{43} \quad (=0.186)$

17.

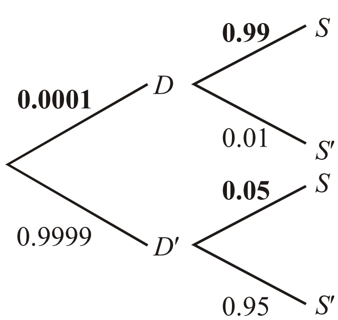


(M1)(A1)

(a) (i) $\frac{2}{3}\frac{2}{3}=\frac{4}{9}$ (ii) $1-\frac{4}{9}=\frac{5}{9}$ or from tree diagram $\frac{2}{9}+\frac{1}{3}=\frac{5}{9}$

(b) Required probability $= \dfrac{\frac{2}{9}}{\frac{2}{9}+\frac{1}{3}} = \frac{2}{5}$.

18. Let D be the event that the patient has the disease and S be the event that the new blood test shows that the patient has the disease. Let D' be the complement of D, *i.e.* the patient does not have the disease.



Therefore $p(S) = 0.0001 \times 0.99 + 0.9999 \times 0.05 = 0.0500939$

$p(D \mid S) = \dfrac{0.0001 \times 0.99}{0.0500939} = 0.00198$ (3 s.f.)

19. $P(4 \text{ girls}) = \left(\frac{13}{24}\times\frac{12}{23}\times\frac{11}{22}\times\frac{10}{21}\right) = \frac{17160}{255024}\left(=\frac{65}{966}=0.0673\right)$

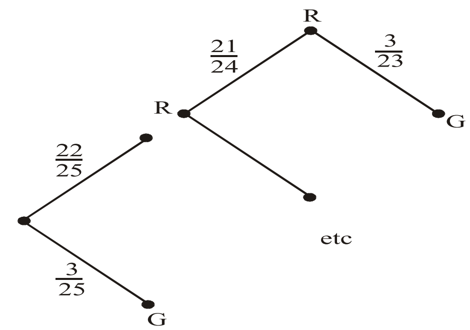
20. $P(RR) = \frac{7}{12} \times \frac{6}{11} \left(= \frac{7}{22} \right)$

$P(YY) = \frac{5}{12} \times \frac{4}{11} \left(= \frac{5}{33} \right)$

P (same colour) = P(RR) + P(YY) = $\frac{31}{66}$ (= 0.470, 3 sf)

21. (a) $P = \frac{22}{23}$ (= 0.957 (3 sf))

(b)



P = P (RRG) + P (RGR) + P (GRR)

$$\frac{22}{25} \times \frac{21}{24} \times \frac{3}{23} + \frac{22}{25} \times \frac{3}{24} \times \frac{21}{23} + \frac{3}{25} \times \frac{22}{24} \times \frac{21}{23} = \frac{693}{2300}$$ (= 0.301 (3 sf))

22. P(different colours) = 1 – [P(GG) + P(RR) + P(WW)]

$= 1 - \left(\frac{10}{6} \times \frac{9}{25} + \frac{10}{26} \times \frac{9}{25} + \frac{6}{26} \times \frac{5}{25} \right) = 1 - \left(\frac{210}{650} \right) = \frac{44}{65}$ (= 0.677, to 3 sf)

OR

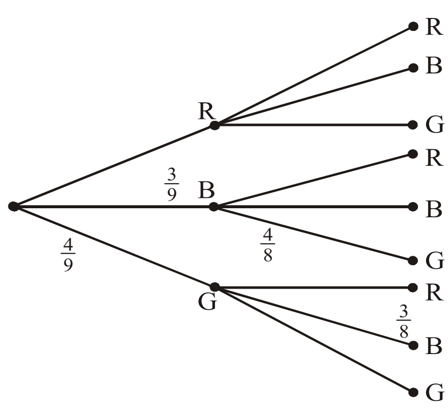
P(different colours) = P(GR) + P(RG) + P(GW) + P(WG) + P(RW) + P(WR)

$= 4\left(\frac{10}{26} \times \frac{6}{25} \right) + 2\left(\frac{10}{26} \times \frac{10}{25} \right) = \frac{44}{65}$ (= 0.677, to 3 sf)

23. (a) $p(GG) = 4/9 \times 3/8 = 1/6$

(b) $p(RR) + P(BB) + P(GG) = 2/9\times1/8 + 3/9\times2/8 + 4/9\times3/8 = 20/72 = 5/18$

(c) Using a tree diagram,



$p(\text{BG or GB}) = \left(\frac{3}{9}\times\frac{4}{8}\right)+\left(\frac{4}{9}\times\frac{3}{8}\right) = \frac{1}{6}+\frac{1}{6}$ **OR** $p(\text{BG or GB}) = 2\times\frac{4}{9}\times\frac{3}{8} = \frac{1}{3}$

24.

Let there be n black disks and $25-n$ white disks.

EITHER

Since the two probabilities are equal,

$$\frac{1}{2}=\frac{\binom{n}{2}+\binom{25-n}{2}}{\binom{25}{2}}$$

$$\frac{1}{2}\binom{25}{2}=\frac{n!}{(n-2)!2!}+\frac{(25-n)!}{(23-n)!2!}$$

$$300=n(n-1)+(25-n)(24-n)$$

$$0=2n^2-50n+300$$

$$0=n^2-25n+150$$

$$0=(n-15)(n-10)$$

$$n=15, 10$$

OR

$$\text{P(same color)}=\frac{n(n-1)}{25\times24}+\frac{(25-n)(24-n)}{25\times24}$$

$$\text{P(different color)}=\frac{n(25-n)}{25\times24}+\frac{(25-n)n}{25\times24}$$

probabilities the same so,

$$\frac{n(n-1)}{25\times24}+\frac{(25-n)(24-n)}{25\times24}=\frac{n(25-n)}{25\times24}+\frac{(25-n)n}{25\times24}$$

$$n^2-n+600-49n+n^2=25n-n^2+25n-n^2$$

$$4n^2-100n+600=0$$

$$n^2-25n+150=0$$

$$(n-10)(n-15)=0$$

$$n=10, 15$$

25. (a) (i) P(Ann wins on her first throw) = $\frac{1}{6}$

(ii) P(Bridget wins on her first throw) = $\frac{5}{6}\times\frac{1}{6}=\frac{5}{36}$

(iii) P(Ann wins on her second throw) = $\frac{5}{6}\times\frac{5}{6}\times\frac{1}{6}=\left(\frac{5}{6}\right)^2\times\frac{1}{6}=\frac{25}{216}$

(b) P(Ann wins) = $\frac{1}{6}+\left(\frac{5}{6}\right)^2\frac{1}{6}+\left(\frac{5}{6}\right)^4\left(\frac{1}{6}\right)+ \ldots \times$

This is an infinite GS with common ratio r = $\left(\frac{5}{6}\right)^2=\frac{25}{36}$

The sum is $S_\infty = \frac{6}{11}$

(c) P(Bridget wins) = $1-\frac{6}{11}=\frac{5}{11}$

26. (a) Probability that Jack wins on his first throw = $\frac{2}{3}$ (or 0.667).

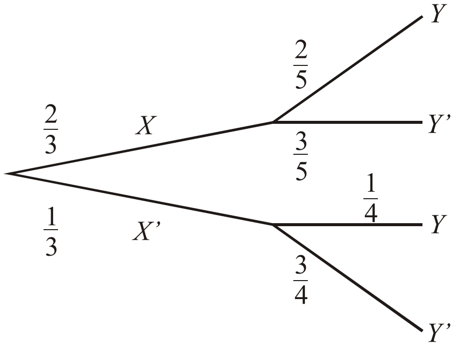
(b) Probability that Jill wins on her first throw: $\frac{1}{3}\times\frac{2}{3}=\frac{2}{9}$ (or 0.222).

(c) **EITHER** Probability that Jack wins the game:

$$\left(\frac{2}{3}\right)+\left(\frac{1}{3}\times\frac{1}{3}\times\frac{2}{3}\right)+\ldots=\frac{2}{3}\times\frac{1}{1-\frac{1}{9}}=\frac{3}{4}$$

B. Paper 2 questions (LONG)

27. (a)

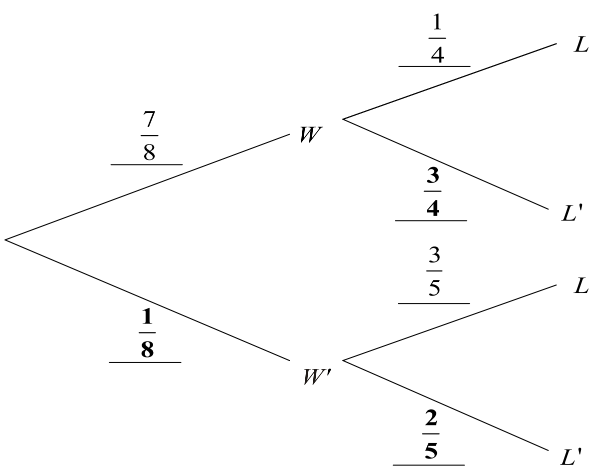


(b) $P(Y') = \frac{2}{3}\times\frac{3}{5}+\frac{1}{3}\times\frac{3}{4} = \frac{13}{20}$

(c) $P(X' \cup Y') = 1 - P(X' \cap Y') = 1 - \frac{4}{15} = \frac{11}{15}$

(d) $P(X' \mid Y') = \dfrac{\frac{1}{3}\times\frac{3}{4}}{\frac{2}{3}\times\frac{3}{5}+\frac{1}{3}\times\frac{3}{4}} = \dfrac{\frac{1}{4}}{\frac{13}{25}} = \frac{25}{52}$

28. (a)



(b) Probability that he will be late is $\frac{7}{8}\times\frac{1}{4}+\frac{1}{8}\times\frac{3}{5}=\frac{47}{160}$ (0.294)

(c) $P(W|L) = \dfrac{P(W \cap L)}{P(L)}$ $\quad P(W \cap L) = \frac{7}{8}\times\frac{1}{4}$ $\quad P(L) = \frac{47}{160}$

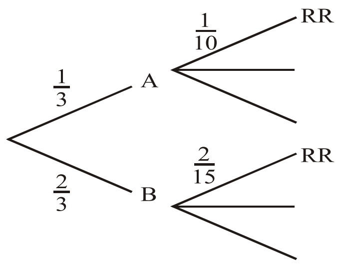
$P(W|L) = \dfrac{\frac{7}{32}}{\frac{47}{160}} = \frac{35}{47}$ (= 0.745)

29. (a) $P(RR) = \left(\frac{2}{5}\right)\left(\frac{1}{4}\right) = \frac{1}{10}$

(b) $P(RR) = \frac{4}{4+n} \times \frac{3}{3+n} = \frac{2}{15}$

$\Leftrightarrow 12 \times 5 = 2\,(4 + n)(3 + n) \Leftrightarrow\ 12 + 7n + n^2 = 90\ \Rightarrow n^2 + 7n - 78 = 0 \Rightarrow n = 6$

(c)



$P(RR) = \frac{1}{3} \times \frac{1}{10} + \frac{2}{3} \times \frac{2}{15} = \frac{11}{90}$

(d) $P(1 \text{ or } 6) = P(A)$

$P(A|RR)\ = \frac{P(A \cap RR)}{P(RR)} = \frac{\left[\left(\frac{1}{3}\right)\left(\frac{1}{10}\right)\right]}{\frac{11}{90}} = \frac{3}{11}$